M06/5/FURMA/SP1/ENG/TZ0/XX

22067101

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Monday 15 May 2006 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working.

1. *[Maximum mark: 9]*

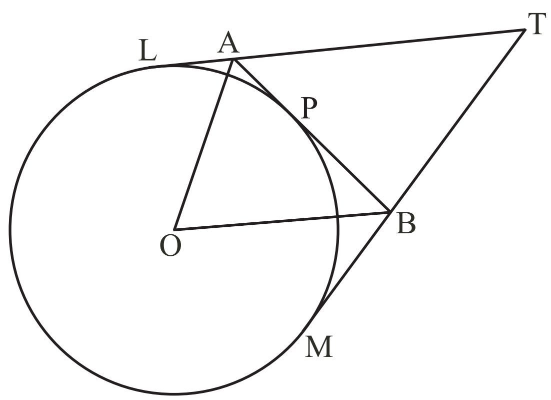
The general term of a sequence is given by the formula $a_n = \dfrac{n^2+3n}{2n^2-1}$, $n \in \mathbb{Z}^+$.

(a) Given that $\lim_{n\to\infty} a_n = L$, where $L \in \mathbb{R}$, find the value of L. *[3 marks]*

(b) Find the smallest value of $N \in \mathbb{Z}^+$ such that $\left| a_n - L \right| < 10^{-3}$ for all $n \geq N$. *[6 marks]*

2. *[Maximum mark: 7]*

The following diagram shows a circle, centre O, and a point T outside the circle. Tangents [TL] and [TM] are drawn to touch the circle at L and M. Let P be any point on the smaller arc LM. The tangent to the circle at P meets [TL] and [TM] at the points A and B respectively.



As P moves around the smaller arc LM, show that A$\hat{\text{O}}$B remains constant. *[7 marks]*

3. *[Maximum mark: 9]*

(a) Convert the base 5 number 2341 to a decimal number. *[3 marks]*

(b) Show that any number written in base 5 is divisible by 2 if the sum of its digits is divisible by 2. *[6 marks]*

4. *[Maximum mark: 11]*

The function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ is defined by $f(x) = \gcd(x, 6)$.

(a) Find the range of the function f. *[3 marks]*

(b) Show that the function f is periodic and find its period. *[3 marks]*

(c) Find the set of positive integers satisfying $f(x) = 2$. *[5 marks]*

5. *[Maximum mark: 12]*

The function f is defined by $f(x) = \begin{cases} 0.005\,\mathrm{e}^{-0.005x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

(a) Show that the function f is a probability density function. *[4 marks]*

(b) While testing the lifetime of light bulbs, in a sample of 150 light bulbs, the following frequency distribution is obtained.

lifetime (hours)	[0, 100[	[100, 200[	[200, 300[	[300, $+\infty$ [
number of light bulbs	47	40	35	28

Use a χ^2 test at the 5 % significance level to determine whether or not the probability distribution defined by f is an appropriate model for the data. *[8 marks]*

6. *[Maximum mark: 12]*

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 + y^2}{xy}$ where $x,\ y > 0$.

(a) Show that the differential equation is homogeneous. *[2 marks]*

(b) Find the general solution of the differential equation, giving your answer in the form $y^2 = f(x)$. *[7 marks]*

(c) Solve the differential equation, given that $y = 2$ when $x = 1$. *[3 marks]*